

# Supplementary Material for “Learning to Transfer for Evolutionary Multitasking”

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## NOMENCLATURE

BBOB	Black-box Optimization Benchmark
DE	Differential Evolution
EMT	Evolutionary Multitasking
GA	Genetic Algorithm
KT	Knowledge Transfer
L2T	Learning to Transfer
MTOP	Multi-Task Optimization Problem

## S.I. LITERATURE REVIEW

### A. Knowledge Transfer for Evolutionary Multitasking

According to the literature review on KT [1], [2], there are two key concerns when designing the KT process for implicit EMT, namely when to transfer and how to transfer. Existing implicit EMT literature has paid research focus on addressing at least one of them and will be briefly reviewed in this subsection.

As the first issue of the implicit KT process, when to transfer refers to deciding whether to trigger a KT process when reproducing each offspring. The KT intensity is a concept that describes the frequency of performing KT operations over a time window of the implicit EMT process. The frequency of performing KT can be controlled by an intensity parameter in a deterministic or probabilistic manner. There are two branches of KT designs on this issue, including fixed-intensity KT and adaptive-intensity KT. The early KT processes are mainly fixed-intensity and the probability of performing KT is unchanged with time, where the intensity-related parameter is usually manually specified by the user before the algorithm running. A representative approach is the multifactorial evolutionary algorithm (MFEA) proposed by Gupta et al. [3]. In MFEA, a random mating probability  $rmp$  is defined to control the probability of performing crossover between solutions from different tasks and is fixed along the implicit EMT process. Later, researchers realized that the KT intensity may need to be adjusted according to the evolution status and the level of inter-task synergy. Then researchers began to design adaptive-intensity KT. Bali et al. [4] proposed a mixture sampling model coefficient to estimate the transferability of source tasks to dynamically control  $rmp$  online. Chen et al. [5] proposed a feedback-based strategy by reinforcing the transfer intensity when the KT between tasks brings positive results. Liang [6] et al. proposed to control the frequency of the KT according to the convergence state estimated based on the fitness difference at previous generations.

As the second issue of the implicit KT process, how to transfer refers to the process of extracting the truly useful materials from the source tasks that benefit the target task. In

implicit EMT, this knowledge extraction process is realized by evolution operators. To this end, researchers have considered characteristics of the specific EC solvers such as DE [7], genetic algorithms (GA) [8], and evolution strategies [9] to design compatible evolution operators for implicit KT. For the implicit EMT algorithms using DE as the base solver, Feng et al. [10] proposed a mutation operator by transferring the differential vector of the source tasks to transfer knowledge. Jin et al. [11] explore the transfer of elite solutions from the source task as the base vector in the mutation process of DE to improve transfer quality. Besides, Chen et al. [5] proposed to use the binomial crossover to exchange dimensions between solutions from two tasks. For the implicit EMT algorithms using GA as the base solver, the simulated binary crossover [3] is a commonly used operator in the literature. Wang et al. [12] developed an implicit KT process that directly transfers the sampled solutions by the GA operator performed on the source population with an anomaly detection strategy. Besides, Zhou et al. [8] proposed to ensemble multiple genetic operators to perform KT, making use of the complementarity of different evolution operators. Due to the distribution discrepancy between source and target tasks, it is found that performing a domain transformation on the source solutions before the execution of the evolution operator of KT would be beneficial [13].

## S.II. PSEUDOCODE OF IMPLEMENTATION OF LEARNING TO TRANSFER

The rollout process by the agent with base solver Genetic Algorithm (GA) is given in **Algorithm S.1**. The pseudocode of the Multi-Task Differential Evolution with Learning to Transfer (MTDE-L2T) equipped with the learned agent is given in **Algorithm S.2**. The pseudocode of MTGA-L2T equipped with the learned agent is given in **Algorithm S.3**.

## S.III. PROBLEM SETUP AND PARAMETER CONFIGURATION

The research objective in this section is to evaluate the adaptability of EMT algorithms, which is measured by the average optimization performance over many unseen MTOP instances. Existing benchmarks such as CEC17MTOP [14] with 9 MTOP instances and CEC19MTOP [15] with 10 MTOP instances can not satisfy our experimental requirements due to the very limited size of the MTOP instance set. Therefore, we propose a new test suite to evaluate the adaptability of EMT algorithms.

**Task instance:** In this study, an optimization task instance  $g(x)$  is in the form of

$$g(x; f, M, x_O) = f(M(x - x_O)) \quad (1)$$

**Algorithm S.1:** Agent rollout with base solver  $GA$ 


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**Input:** Task pair  $\mathcal{T}=\{f_1, f_2\}$ , base solver  $\mathcal{A} = GA$ , maximum generations  $G_{\text{roll}}$ , parameterized agent  $\pi(s|\theta)$ , initial population set  $\mathcal{P} = \{P_1, \dots, P_{N_P}\}$ , population size per task  $N$

**Output:** Experience data buffer  $\mathcal{D}$

- 1  $\mathcal{D} = \emptyset$ ; // Empty rollout data buffer
- 2 Initialize population  $X$  of size  $N$  for each task by randomly selecting initial population from  $\mathcal{P}$ ;
- 3 Evaluate fitness of  $X$  to obtain  $Y$  for each task;
- 4 Calculate initial state  $s$  by concatenating  $O_c$  and  $O_t$ ;
- 5  $g = 0$ ;
- 6 **while**  $g < G_{\text{max}}$  **do**
- 7    $a = \pi(s|\theta)$ ; // predict action by actor network
- 8    $P_m = X_1 \cup X_2$ ; // merge populations of two tasks
- 9   Empty the offspring set  $U_1, U_2$  for two tasks;
- 10   Retrieve KT action parameters  $a_{k,1}, a_{k,2}, a_{k,3}$  for task  $f_k$  from  $a$ ;
- 11   **while** number of offspring for each task  $< N$  **do**
- 12     Sample two individuals  $p_a, p_b$  from  $P_m$  as parents;
- 13      $P_m = P_m - \{p_a, p_b\}$ ;
- 14     Get the associated task indices  $k_a, k_b$  of  $p_a, p_b$ ;
- 15     **if**  $k_a == k_b$  **then**
- 16       // belong to the same task
- 17       Perform crossover and mutation on  $p_a, p_b$  to obtain two offsprings  $u_a, u_b$  for task  $f_k$ ;
- 18     **else if**  $\text{rand} < a_{k_a,1}$  **then**
- 19       // perform KT between tasks
- 20       Sample  $u_a$  by the proposed action formulation with parameters  $a_{k_a,2}, a_{k_a,3}$ ;
- 21       Perform mutation on  $p_b$  to obtain  $u_a$ ;
- 22     **else**
- 23       Perform mutation on  $p_a, p_b$  to obtain  $u_a, u_b$ ;
- 24     **end**
- 25     Add  $u_a, u_b$  to their corresponding offspring set  $U_1, U_2$ ;
- 26   **end**
- 27   Evaluate fitness of  $U_1, U_2$  on  $f_1, f_2$  respectively;
- 28   **foreach** task  $f_k$  **do**
- 29     Calculate task-specific features  $O_{t,k}$  of task  $f_k$ ;
- 30     Calculate reward  $r_k$ ;
- 31     Update population  $POP_k$  by selection;
- 32   **end**
- 33    $r = r_1 + r_2$ ; // Sum up rewards of the tasks
- 34   Calculate common features  $O_c$ ;
- 35   Update state  $s$  by concatenating  $O_c$  and  $O_t$ ;
- 36    $\mathcal{D} = \mathcal{D} \cup (s, a, r)$ ;
- 37    $g = g + 1$ ;
- 38 **end**

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where  $f$  denotes a function class represented by mathematical formula,  $M$  denotes the rotation matrix, and  $x_O$  denotes the shift vector. Here,  $M$  rotates the search landscape, introducing variable correlations, while  $x_O$  shifts the optimum. Note that we assume the optimization task to be black-box, meaning the structure information of the function like first-order gradient is inaccessible to the algorithms. Herein, we denote the set containing different function classes  $f$  as  $\mathcal{F}$  and the set containing different parametric configurations  $s : (M, x_O)$  as the configuration set  $\mathcal{S}$ .

**Task instance set:** The task instance set  $\Theta$  is used to construct MTOP instances. A task instance set  $\Theta$  is defined

**Algorithm S.2:** MTDE-L2T

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**Input:** Task pair  $\mathcal{T}=\{f_1, f_2\}$ , maximum generations  $G_{\text{max}}$ , learned agent  $\pi(s|\theta^*)$ , initial population set  $\mathcal{P} = \{P_1, \dots, P_{N_P}\}$ , population size per task  $N$

**Output:** Best-found solutions for two tasks  $x_1^*, x_2^*$

- 1 Initialize population  $X$  of size  $N$  for each task by randomly selecting initial population from  $\mathcal{P}$ ;
- 2 Evaluate fitness of  $X$  to obtain  $Y$  for each task;
- 3 Calculate initial state  $s$  by concatenating  $O_c$  and  $O_t$ ;
- 4  $g = 0$ ;
- 5 **while**  $g < G_{\text{max}}$  **do**
- 6    $a = \pi(s|\theta^*)$ ; // predict action by actor network
- 7   **foreach** task  $f_k$  **do**
- 8     Retrieve KT action parameters  $a_{k,1}, a_{k,2}, a_{k,3}$  for task  $f_k$  from  $a$ ;
- 9     Sample offspring population  $U$  by base solver  $DE$ ;
- 10      $N_{KT} = \lceil 0.5 \cdot a_{k,1} \rceil$ ;
- 11     Randomly select  $N_{KT}$  indices from  $\{1, \dots, N\}$  to construct a index set  $\mathcal{I}_{KT} = \{j_1, \dots, j_{N_{KT}}\}$ ;
- 12     **foreach** index  $j$  in  $\mathcal{I}_{KT}$  **do**
- 13       Sample  $v_{k,j}$  by the proposed action formulation with KT action parameters  $a_{k,2}, a_{k,3}$ ;
- 14       Perform binomial crossover to obtain  $u_{k,j}$ ;
- 15       Replace  $j$ -th solution in  $U$  with  $u_{k,j}$ ;
- 16     **end**
- 17     Evaluate fitness  $Y_k$  of  $U$  on  $f_k$ ;
- 18     Calculate task-specific features  $O_{t,k}$  of task  $f_k$ ;
- 19     Update population  $POP_k$  by selection;
- 20   **end**
- 21   Calculate common features  $O_c$ ;
- 22   Update state  $s$  by concatenating  $O_c$  and  $O_t$ ;
- 23   Update best-found solutions  $x_1^*, x_2^*$ ;
- 24    $g = g + 1$ ;
- 25 **end**

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by the product of a function class set  $\mathcal{F}$  and a configuration set  $\mathcal{S}$ , i.e.,  $\Theta = \mathcal{F} \times \mathcal{S}$ . Hence, the task instance set's size is  $|\Theta| = |\mathcal{F}| \times |\mathcal{S}|$ . There are two ways to configure a task instance set either by (1) parameterization or (2) specification. In the parameterization, the task instance set  $\Theta$  is configured by parameterizing the task e.g., by  $(M, x_O)$  in a continuous space and defining a distribution on this space. In the specification, the task instance set  $\Theta$  can be constructed by manually specifying a number of task instances. With the definition of  $\Theta$ , we have explicitly or implicitly defined the distribution of  $g$  as a random variable  $G$ , i.e.,

$$g(x) \sim p(G; \Theta). \quad (2)$$

**MTOP instance:** An MTOP instance, denoted as  $\mathcal{T}$ , for the EMT algorithm to solve is a pair or a set of task instances, which is defined as

$$\mathcal{T} = \{g_k(x)\}, \mathcal{X}_k \subseteq \mathbb{R}^{D_k}, k = 1, 2, \dots, K, \quad (3)$$

where  $K$  is the number of tasks,  $\mathcal{X}_k$  and  $D_k$  are the solution space and the dimensionality of the solution for task  $g_k(x)$ , respectively. Without loss of generality, only the box constraints will be studied, i.e.,  $\mathcal{X}_k = [L_k, U_k]^{D_k}$ , where  $L_k$  and  $U_k$  are the lower and upper boundaries of the solution space respectively. The objective of an MTOP instance is to

**Algorithm S.3:** MTGA-L2T

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**Input:** Task pair  $\mathcal{T}=\{f_1, f_2\}$ , maximum generations  $G_{\max}$ , learned agent  $\pi(s|\theta^*)$ , initial population set  $\mathcal{P} = \{P_1, \dots, P_{N_P}\}$ , population size per task  $N$

**Output:** Best-found solutions for two tasks  $x_1^*, x_2^*$

- 1 Initialize population  $X$  of size  $N$  for each task by randomly selecting initial population from  $\mathcal{P}$ ;
- 2 Evaluate fitness of  $X$  to obtain  $Y$  for each task;
- 3 Calculate initial state  $s$  by concatenating  $O_c$  and  $O_t$ ;
- 4  $g = 0$ ;
- 5 **while**  $g < G_{\max}$  **do**
- 6      $a = \pi(s|\theta^*)$ ; // predict action by actor network
- 7      $P_m = X_1 \cup X_2$ ; // merge populations of two tasks
- 8     Empty the offspring set  $U_1, U_2$  for two tasks;
- 9     Retrieve KT action parameters  $a_{k,1}, a_{k,2}, a_{k,3}$  for task  $f_k$  from  $a$ ;
- 10    **while** number of offspring for each task  $< N$  **do**
- 11     Sample two individuals  $p_a, p_b$  from  $P_m$  as parents;
- 12      $P_m = P_m - \{p_a, p_b\}$ ;
- 13     Get the associated task indices  $k_a, k_b$  of  $p_a, p_b$ ;
- 14     **if**  $k_a == k_b$  **then**
- 15         // belong to the same task
- 16         Perform crossover and mutation on  $p_a, p_b$  to obtain two offsprings  $u_a, u_b$  for task  $f_k$ ;
- 17     **else if**  $\text{rand} < a_{k_a,1}$  **then**
- 18         // perform KT between tasks
- 19         Sample  $u_a$  by the proposed action formulation with parameters  $a_{k_a,2}, a_{k_a,3}$ ;
- 20         Perform mutation on  $p_b$  to obtain  $u_a$ ;
- 21     **else**
- 22         Perform mutation on  $p_a, p_b$  to obtain  $u_a, u_b$ ;
- 23     **end**
- 24     Add  $u_a, u_b$  to their corresponding offspring set  $U_1, U_2$ ;
- 25    **end**
- 26    Evaluate fitness of  $U_1, U_2$  on  $f_1, f_2$  respectively;
- 27    **foreach** task  $f_k$  **do**
- 28     Calculate task-specific features  $O_{t,k}$  of task  $f_k$ ;
- 29     Calculate reward  $r_k$ ;
- 30     Update population  $POP_k$  by selection;
- 31    **end**
- 32    Calculate common features  $O_c$ ;
- 33    Update state  $s$  by concatenating  $O_c$  and  $O_t$ ;
- 34    Update best-found solutions  $x_1^*, x_2^*$ ;
- 35     $g = g + 1$ ;
- 36 **end**

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find the optimal solution for each task, and in the case of minimization can be represented as

$$x_k^* = \arg \min_{x \in \mathcal{X}_k} g_k(x), k = 1, 2, \dots, K. \quad (4)$$

The solution spaces of tasks may not be identical and have different lower and upper boundaries. To allow KT between different tasks, a common strategy [3] is to perform an linear transformation  $(x_k - L_k)/(U_k - L_k)$  on  $x_k$  to encode the solutions to a unified search space  $\mathcal{U}_k = [0, 1]^{D_u}$ , where  $D_u = \max \{D_k\}$ . An MTOP instance  $\mathcal{T}$  with  $K$  tasks is constructed by selecting or sampling  $K$  pairs  $(f, s)$  from a task instance set  $\Theta$ . In this study, we assume all tasks of an MTOP instance are drawn independently from the same distribution, i.e.,

$$p(\mathcal{T}; \Theta) = p(g_1, \dots, g_K; \Theta) = \prod_{k=1}^K p(G = g_k; \Theta) \quad (5)$$

**MTOP instance set:** With the definition of MTOP instance  $\mathcal{T}$ , we can now define an MTOP instance set  $\Gamma = \{\mathcal{T}_j\}_{j=1}^{N_\Gamma}$  containing  $N_\Gamma$  different MTOP instances that are randomly sampled based on the predefined task instance set  $\Theta$ . Without loss of generality, we assume all MTOP instances are drawn from the same distribution  $\mathcal{T}_j \sim p(\mathcal{T}; \Theta), j = 1, \dots, N_\Gamma$ . Next, we can define multiple task instance sets  $\Theta_1, \Theta_2, \dots$  by configuring  $\mathcal{F}_1, \mathcal{F}_2, \dots$  and  $\mathcal{S}_1, \mathcal{S}_2, \dots$  respectively to construct multiple MTOP instance sets  $\Gamma_1, \Gamma_2, \dots$ . Different MTOP instance sets represent different distributions for learning the agent and testing the performance of the EMT algorithms. In the experiment, we train an agent on a specific MTOP set and test the performance on multiple MTOP sets.

### A. Problem Setting

To define task instances, we employ two sets of synthetic functions. The first function set  $\mathcal{F} = \{\text{Ackley}, \text{Griewank}, \text{Rastrigin}, \text{Sphere}, \text{Weierstrass}\}$  includes the functions from the CEC17MTOP benchmark [14] with highly configurable global optimum by varying  $x_0$ . The functions' solution space is normalized to  $\mathcal{X} = [0, 1]^D$  based on the lower bound and upper bound of each function [16]. Based on this function set  $\mathcal{F}$ , we build different task instance sets  $\Theta$  by defining different task optimum distributions  $p(x_0)$ , thereby obtaining multiple MTOP instance sets. The configurations of MTOP instance sets are given in Table S.I. The shift  $x_0$  lies in the search space  $[0, 1]^D$ ,  $C$  is the number of clusters,  $x_{c,i}$  is the center of the  $i$ -th cluster, and  $\Delta_i$  is the radius of the  $i$ -th cluster. Specifically, we define 10 MTOP sets with different characteristics in the task optimum distribution range (i.e., VS, S, M, L, and VL) and the number of distribution clusters (i.e., C1-C5). Note that since we directly define the distribution of  $p(x_0)$  in a continuous space, the cardinality of the configuration set  $\mathcal{S}$  and produced task instance set  $\Theta$  is infinite. For each MTOP set in Table S.I, we learn an agent and test the learned agent on the problem set independently. That is, we obtain 10 learned agents for 10 problem sets, respectively. Therefore, the training MTOP instances and testing MTOP instances are independent and identically distributed (i.i.d.), allowing us to assess the effectiveness of L2T in adapting to different MTOP distributions of interest.

Different from the first function set, the second function set includes a broader spectrum of functions with diverse characteristics from the black-box optimization benchmark (BBOB) [17]. BBOB is a widely used benchmark for evaluating and comparing black-box optimizers. The BBOB contains a total of 24 classes of functions to serve as optimization tasks, denoted as  $\{f_1, \dots, f_{24}\}$ , with different landscape properties which can be categorized into five groups, namely separable functions  $\{f_1, \dots, f_5\}$ , functions with low or moderate conditioning  $\{f_6, \dots, f_9\}$ , functions with high conditioning and unimodal  $\{f_{10}, \dots, f_{14}\}$ , multi-modal functions with adequate global structure  $\{f_{15}, \dots, f_{19}\}$ , and multi-modal functions with weak global structure  $\{f_{20}, \dots, f_{24}\}$ . The function ID is denoted as  $fid \in \{1, \dots, 24\}$  and  $(M, x_0)$  of the task instances in BBOB is generated using a random number generator a

with seed ID denoted as  $sid$ . Hence, each task instance in BBOB is defined as a tuple  $(fid, sid)$ . This setup enables BBOB to create numerous task instances by varying the function ID  $fid$  and the seed  $sid$ . The functions' search space is set to  $\mathcal{X} = [-5, 5]^D$ . The optimums of most functions are uniformly distributed in a wide range  $[-4, 4]^D$ , posing challenges to EMT algorithms' adaptability. For clarity, we define the function ID set as  $\mathcal{F}$  containing different  $fid$  and the seed set as  $\mathcal{S}$  containing different  $sid$ .

For the BBOB, we formulate 16 MTOP instance sets with their corresponding configurations on  $\mathcal{F}$  and  $\mathcal{S}$  shown in Table S.II. The 16 MTOP instance sets contain one set for learning (i.e., BBOB<sub>learn</sub>) and 15 sets (i.e., BBOB1-BBOB15) with unseen MTOP instances for testing. That is, we obtain one learned agent and test it on the remaining 15 MTOP sets. Regarding the MTOP set for learning, we want to cover different kinds of complex functions for the agent to learn versatile KT skills for handling diverse problems. Therefore, we select functions  $\{f_1, f_3, f_8, f_{10}, f_{16}, f_{20}\}$  from each of the five function groups in BBOB. We categorize the test MTOP instance sets into two groups to assess adaptability: one for evaluating nearly i.i.d. scenarios and another for potential out-of-distribution (o.o.d.) cases. The nearly i.i.d. group contains MTOP instances with the same functions as the learning phase but varies in real distribution coverage (BBOB1 and BBOB2) and function-type weighting (BBOB3-BBOB8), aligning closely with the training set BBOB<sub>learn</sub>. Conversely, the o.o.d. group comprises entirely new functions not encountered during learning (BBOB9-BBOB15), with BBOB9 presenting a substantial challenge by including all functions omitted in the learning stage. To evaluate adaptability to unseen yet similar functions, we create six MTOP sets (BBOB10-BBOB15) using selected functions  $f_2, f_6, f_{12}, f_{15}, f_{21}$  from BBOB's five function groups. This problem setting allows us to evaluate the generalization ability of the learned agent. A notable difference between BBOB-based MTOP sets and the ones based on CEC19MTOP is that the number of training instances in BBOB-based MTOP sets is limited, which is a more practical situation faced in the real world.

### B. Parameter Configuration

The parameters of the proposed L2T framework are given in Table S.I. The number of tasks of the MTOP instance is set to  $K = 2$ . For the learning stage, the maximum number of rollout generations is set as  $G_{roll} = 100$  and the target accuracy is  $\xi = 1e-8$ . For the testing stage, we set the maximum number of generations as  $G_{max} = 250$  which is larger than the rollout generations  $G_{roll} = 100$ . The parameters  $\beta_1, \beta_2$ , and  $\beta_3$  in the reward function are set to  $\beta_1 = 1, \beta_2 = 10$  and  $\beta_3 = G_{roll} = 100$ , respectively. Each problem set for the testing stage contains  $N_{\Gamma} = 100$  randomly sampled MTOP instances.

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TABLE S.I  
CONFIGURATIONS OF MTOP INSTANCE SET WITH DIFFERENT TASK OPTIMUM DISTRIBUTION BASED ON CEC17MTOP

MTOP set	Task optimum distribution configuration ( $S$ )
VS	$x_o \sim U[0.5 - \Delta, 0.5 + \Delta]^D, \Delta = 0.025$
S	$x_o \sim U[0.5 - \Delta, 0.5 + \Delta]^D, \Delta = 0.05$
M	$x_o \sim U[0.5 - \Delta, 0.5 + \Delta]^D, \Delta = 0.1$
L	$x_o \sim U[0.5 - \Delta, 0.5 + \Delta]^D, \Delta = 0.2$
VL	$x_o \sim U[0.5 - \Delta, 0.5 + \Delta]^D, \Delta = 0.4$
C1	$x_o \sim x_{c,1} + L(x_{c,2} - x_{c,1}), L \sim U[0, 1], x_{c,1}, x_{c,2} \in [0, 1]^D$
C2	$x_o \sim U[x_{c,i} - \Delta_i, x_{c,i} + \Delta_i   Z = z_i]^D, Z \sim p(Z) = 1/C, C = 2$
C3	$x_o \sim U[x_{c,i} - \Delta_i, x_{c,i} + \Delta_i   Z = z_i]^D, Z \sim p(Z) = 1/C, C = 3$
C4	$x_o \sim U[x_{c,i} - \Delta_i, x_{c,i} + \Delta_i   Z = z_i]^D, Z \sim p(Z) = 1/C, C = 4$
C5	$x_o \sim U[x_{c,i} - \Delta_i, x_{c,i} + \Delta_i   Z = z_i]^D, Z \sim p(Z) = 1/C, C = 5$

TABLE S.II  
CONFIGURATIONS OF MTOP INSTANCE SET BASED ON BBOB

Use purpose	MTOP set	Task function ID set ( $\mathcal{F}$ )	Task seed set ( $S$ )
For learning	BBOB <sub>learn</sub>	{1, 3, 8, 10, 16, 20}	[1,100]
For testing nearly i.i.d. adaptability	BBOB1	{1, 3, 8, 10, 16, 20}	[500,1500]
	BBOB2	{1, 3, 8, 10, 16, 20}	[1000,1005]
	BBOB3	{1}	[500,1500]
	BBOB4	{3}	[500,1500]
	BBOB5	{8}	[500,1500]
	BBOB6	{10}	[500,1500]
	BBOB7	{16}	[500,1500]
	BBOB8	{20}	[500,1500]
For testing o.o.d. adaptability	BBOB9	{1, ..., 24} -	[500,1500]
	BBOB10	{1, 3, 8, 10, 16, 20}	[500,1500]
	BBOB11	{2, 6, 12, 15, 21}	[500,1500]
	BBOB12	{2}	[500,1500]
	BBOB13	{6}	[500,1500]
	BBOB14	{12}	[500,1500]
	BBOB15	{15}	[500,1500]

TABLE S.III  
PARAMETER CONFIGURATION OF THE PROPOSED L2T FRAMEWORK IN THE EXPERIMENTAL STUDIES

Parameter	Value
Maximum rollout generations $G_{roll}$	$G_{roll} = 100$
Maximum generations for testing $G_{max}$	$G_{max} = 250$
Target accuracy $\xi$	$\xi = 1e - 8$
Actor-critic network structure	Two hidden layers and one linear layer, # hidden neurons=64, activation function is tanh( $\cdot$ )
Population size $N$	$N = 50$
Evolution operators for DE	DE/rand/1 mutation and binomial crossover
DE-related parameters	$F = 0.5, CR = 0.5$
Evolution operators for GA	Simulated binary crossover (SBX) and polynomial mutation (PM)
GA-related parameters	$\eta_c = 2, \eta_m = 5$
Reward function coefficients $\beta_1, \beta_2, \beta_3$	$\beta_1 = 1, \beta_2 = 10, \beta_3 = G_{roll}$
PPO-related parameters	$\gamma = 0.99, \lambda = 0.95, \epsilon = 0.2$
Initial population set size $N_P$	$N_P = 10$
Number of parallel environments $N_{env}$	$N_{env} = 20$
Rollout data buffer size $N_{buff}$	$N_{buff} = 2048 * N_{env} = 40960$
Maximum number of time steps for learning $T$	$T = 5e6$ for BBOB <sub>learn</sub> and $T = 2e6$ for VS, S, M, L, VL, C1, C2, C3, C4, and C5

TABLE S.IV  
COMPARATIVE RESULTS BETWEEN MTDE-L2T AND OTHER IMPLICIT EMT ALGORITHMS WITH AGENTS LEARNED SEPARATELY ON DIFFERENT PROBLEM SETS AT GENERATION= $G_{roll}$

Problem	AEMTO	MFDE	MKTDE	MTDE-AD	MTDE-B
VS	100/0/0(+)	100/0/0(+)	86/7/7(+)	99/1/0(+)	97/3/0(+)
S	94/6/0(+)	88/8/4(+)	66/15/19(+)	88/11/1(+)	93/7/0(+)
M	87/13/0(+)	72/15/13(+)	50/17/33(+)	84/14/2(+)	82/16/2(+)
L	79/19/2(+)	51/29/20(+)	46/24/30(+)	71/26/3(+)	46/49/5(+)
VL	50/48/2(+)	31/47/22(+)	35/34/31(+)	45/54/1(+)	2/92/6(-)
C1	74/26/0(+)	61/29/10(+)	44/30/26(+)	63/36/1(+)	40/58/2(+)
C2	47/48/5(+)	51/33/16(+)	32/26/42(-)	34/59/7(+)	16/75/9(+)
C3	35/56/9(+)	37/42/21(+)	34/30/36(-)	34/51/15(+)	9/87/4(+)
C4	42/44/14(+)	43/39/18(+)	46/21/33(+)	35/51/14(+)	9/83/8(+)
C5	45/48/7(+)	37/46/17(+)	45/27/28(+)	44/46/10(+)	7/83/10(-)

TABLE S.V  
COMPARATIVE RESULTS BETWEEN THE PROPOSED L2T-BASED AND OTHER IMPLICIT EMT ALGORITHMS AT GENERATION= $G_{\max}$

Problem	MTDE-L2T vs					MTGA-L2T vs				
	AEMTO	MFDE	MKTDE	MTDE-AD	MTDE-B	GMFEA	MFEA	MFEA-AKT	MFEA2	MTEA-AD
BBOB1	35/55/10(+)	44/52/4(+)	59/26/15(+)	40/50/10(+)	29/61/10(+)	66/26/8(+)	64/30/6(+)	66/27/7(+)	62/35/3(+)	54/23/23(+)
BBOB2	41/54/5(+)	44/50/6(+)	66/28/6(+)	40/53/7(+)	26/60/14(+)	63/30/7(+)	65/30/5(+)	65/31/4(+)	67/31/2(+)	63/14/23(+)
BBOB3	0/100/0(=)	0/100/0(=)	0/100/0(=)	7/93/0(+)	0/100/0(=)	100/0/0(+)	100/0/0(+)	100/0/0(+)	100/0/0(+)	100/0/0(+)
BBOB4	67/32/1(+)	10/85/5(+)	100/0/0(+)	57/43/0(+)	37/62/1(+)	100/0/0(+)	100/0/0(+)	100/0/0(+)	100/0/0(+)	100/0/0(+)
BBOB5	1/88/11(-)	3/80/17(-)	3/79/18(-)	2/92/6(-)	4/88/8(-)	51/44/5(+)	62/33/5(+)	58/36/6(+)	39/59/2(+)	46/49/5(+)
BBOB6	30/70/0(+)	38/62/0(+)	81/19/0(+)	30/69/1(+)	1/81/18(-)	8/71/21(-)	7/72/21(-)	9/77/14(-)	28/68/4(+)	6/64/30(-)
BBOB7	4/93/3(+)	3/92/5(-)	2/95/3(-)	2/92/6(-)	4/89/7(-)	11/82/7(+)	13/78/9(+)	14/83/3(+)	23/74/3(+)	0/29/71(-)
BBOB8	8/86/6(+)	3/93/4(-)	73/27/0(+)	10/87/3(+)	6/88/6(=)	29/70/1(+)	29/71/0(+)	16/80/4(+)	16/81/3(+)	58/42/0(+)
BBOB9	41/53/6(+)	36/45/19(+)	54/28/18(+)	41/53/6(+)	26/59/15(+)	51/37/12(+)	59/32/9(+)	56/29/15(+)	65/25/10(+)	54/20/26(+)
BBOB10	60/32/8(+)	36/41/23(+)	40/30/30(+)	68/25/7(+)	36/41/23(+)	77/13/10(+)	80/16/4(+)	79/16/5(+)	80/14/6(+)	58/13/29(+)
BBOB11	96/0/4(+)	12/88/0(+)	2/93/5(-)	96/0/4(+)	9/91/0(+)	99/0/1(+)	99/0/1(+)	99/0/1(+)	99/0/1(+)	97/0/3(+)
BBOB12	33/61/6(+)	0/4/96(-)	0/9/91(-)	40/58/2(+)	30/65/5(+)	97/0/3(+)	99/0/1(+)	100/0/0(+)	100/0/0(+)	98/1/1(+)
BBOB13	6/80/14(-)	10/86/4(+)	15/81/4(+)	6/90/4(+)	11/87/2(+)	97/0/3(+)	96/0/4(+)	96/0/4(+)	96/0/4(+)	90/0/10(+)
BBOB14	11/88/1(+)	9/88/3(+)	8/87/5(+)	3/93/4(-)	6/90/4(+)	5/83/12(-)	3/87/10(-)	8/87/5(+)	18/82/0(+)	0/31/69(-)
BBOB15	7/88/5(+)	12/81/7(+)	19/78/3(+)	14/82/4(+)	3/85/12(-)	26/49/25(+)	25/51/24(+)	21/55/24(-)	25/55/20(+)	22/54/24(-)

TABLE S.VI  
COMPARATIVE RESULTS BETWEEN THE PROPOSED L2T-BASED AND EXPLICIT EMT ALGORITHMS AT DIFFERENT GENERATION  $G$

Problem	MTDE-L2T vs MTDE-EA		MTGA-L2T vs ATMFEA	
	$G = G_{\text{roll}}$	$G = G_{\text{max}}$	$G = G_{\text{roll}}$	$G = G_{\text{max}}$
BBOB1	61/39/0(+)	39/52/9(+)	77/15/8(+)	59/31/10(+)
BBOB2	62/37/1(+)	40/53/7(+)	76/18/6(+)	65/30/5(+)
BBOB3	98/2/0(+)	0/100/0(=)	100/0/0(+)	100/0/0(+)
BBOB4	22/77/1(+)	61/39/0(+)	100/0/0(+)	100/0/0(+)
BBOB5	12/84/4(+)	4/91/5(-)	74/15/11(+)	58/32/10(+)
BBOB6	16/80/4(+)	30/68/2(+)	8/63/29(-)	6/70/24(-)
BBOB7	5/94/1(+)	1/89/10(-)	29/69/2(+)	16/78/6(+)
BBOB8	34/66/0(+)	6/90/4(+)	100/0/0(+)	28/72/0(+)
BBOB9	33/61/6(+)	41/49/10(+)	51/30/19(+)	50/22/28(+)
BBOB10	42/41/17(+)	50/38/12(+)	70/22/8(+)	73/14/13(+)
BBOB11	94/1/5(+)	96/1/3(+)	100/0/0(+)	99/0/1(+)
BBOB12	37/62/1(+)	36/61/3(+)	57/37/6(+)	99/0/1(+)
BBOB13	59/38/3(+)	10/82/8(+)	98/0/2(+)	94/0/6(+)
BBOB14	7/93/0(+)	8/87/5(+)	5/85/10(-)	2/55/43(-)
BBOB15	16/78/6(+)	12/77/11(+)	21/55/24(-)	22/51/27(-)

TABLE S.VII  
COMPARATIVE RESULTS OF TRAINED-FROM-SCRATCH AND FINE-TUNED MTDE-L2T AND OTHER IMPLICIT EMT ALGORITHMS AT GENERATION= $G_{\max}$

Problem	MTDE-L2T-w/o-FT vs					
	AEMTO	MFDE	MKTDE	MTDE-AD	MTDE-B	MTDE-EA
VS	97/2/1(+)	77/3/20(+)	56/13/31(+)	93/5/2(+)	92/7/1(+)	96/3/1(+)
S	91/9/0(+)	70/8/22(+)	53/11/36(+)	85/13/2(+)	87/12/1(+)	88/10/2(+)
M	84/15/1(+)	67/9/24(+)	47/15/38(+)	84/13/3(+)	77/21/2(+)	88/11/1(+)
L	79/15/6(+)	59/14/27(+)	49/12/39(+)	79/16/5(+)	45/50/5(+)	82/15/3(+)
VL	49/45/6(+)	27/39/34(-)	28/29/43(-)	43/43/14(+)	3/93/4(-)	48/47/5(+)
C1	69/21/10(+)	53/13/34(+)	35/16/49(-)	68/26/6(+)	37/60/3(+)	69/29/2(+)
C2	44/46/10(+)	41/20/39(+)	26/21/53(-)	45/44/11(+)	22/76/2(+)	56/38/6(+)
C3	41/50/9(+)	28/31/41(-)	31/22/47(-)	42/48/10(+)	10/87/3(+)	46/48/6(+)
C4	41/43/16(+)	35/27/38(-)	37/13/50(-)	38/39/23(+)	7/83/10(-)	52/43/5(+)
C5	43/48/9(+)	31/34/35(-)	34/22/44(-)	46/37/17(+)	6/86/8(-)	53/44/3(+)
BBOB9	21/74/5(+)	36/31/33(+)	49/27/24(+)	29/63/8(+)	4/88/8(-)	16/80/4(+)
BBOB10	53/46/1(+)	34/23/43(-)	33/25/42(-)	48/51/1(+)	10/83/7(+)	52/45/3(+)
Problem	MTDE-L2T-FT vs					
	AEMTO	MFDE	MKTDE	MTDE-AD	MTDE-B	MTDE-EA
VS	87/8/5(+)	82/14/4(+)	41/21/38(+)	91/6/3(+)	78/18/4(+)	91/7/2(+)
S	88/10/2(+)	80/17/3(+)	45/21/34(+)	88/10/2(+)	72/24/4(+)	91/8/1(+)
M	87/9/4(+)	81/15/4(+)	52/17/31(+)	89/9/2(+)	76/23/1(+)	88/9/3(+)
L	89/9/2(+)	75/20/5(+)	52/22/26(+)	87/7/6(+)	69/26/5(+)	89/8/3(+)
VL	72/13/15(+)	55/32/13(+)	51/25/24(+)	67/19/14(+)	47/42/11(+)	80/14/6(+)
C1	68/15/17(+)	62/26/12(+)	44/25/31(+)	69/17/14(+)	56/35/9(+)	83/10/7(+)
C2	53/24/23(+)	49/42/9(+)	28/31/41(-)	49/27/24(+)	34/50/16(+)	61/36/3(+)
C3	55/28/17(+)	39/44/17(+)	43/27/30(+)	61/24/15(+)	35/51/14(+)	69/26/5(+)
C4	65/12/23(+)	50/35/15(+)	38/22/40(-)	59/16/25(+)	50/37/13(+)	75/13/12(-)
C5	63/24/13(+)	48/42/10(+)	53/24/23(+)	65/21/14(+)	40/48/12(+)	65/27/8(+)
BBOB9	40/49/11(+)	33/45/22(+)	54/29/17(+)	40/51/9(+)	29/63/8(+)	39/53/8(+)
BBOB10	67/24/9(+)	35/41/24(+)	36/29/35(+)	63/26/11(+)	39/58/3(+)	66/26/8(+)

TABLE S.VIII  
COMPARATIVE RESULTS BETWEEN THE LEARNED AGENT AND PREDEFINED AGENTS AT GENERATION= $G_{\max}$

Problem	MTDE-f(.5,0,1)	MTDE-f(.5,1,0)	MTDE-f(.5,1,1)	MTDE-f(1,0,1)	MTDE-f(1,1,0)	MTDE-f(1,1,1)	MTDE-r	STDE
VS	76/6/18(+)	97/1/2(+)	99/0/1(+)	74/5/21(+)	100/0/0(+)	98/0/2(+)	92/6/2(+)	95/5/0(+)
S	69/9/22(+)	98/2/0(+)	97/3/0(+)	68/8/24(+)	100/0/0(+)	100/0/0(+)	91/8/1(+)	88/11/1(+)
M	67/9/24(+)	92/7/1(+)	90/9/1(+)	67/9/24(+)	100/0/0(+)	100/0/0(+)	90/8/2(+)	72/25/3(+)
L	57/16/27(+)	84/10/6(+)	80/11/9(+)	57/15/28(+)	91/1/8(+)	92/0/8(+)	75/13/12(+)	44/52/4(+)
VL	28/38/34(-)	57/26/17(+)	52/28/20(+)	26/37/37(-)	81/4/15(+)	78/4/18(+)	43/23/34(+)	7/9/12(+)
C1	51/14/35(+)	75/16/9(+)	80/11/9(+)	51/14/35(+)	93/2/5(+)	89/1/10(+)	66/19/15(+)	46/46/8(+)
C2	32/26/42(-)	52/31/17(+)	51/29/20(+)	37/20/43(-)	70/13/17(+)	66/14/20(+)	53/29/18(+)	24/70/6(+)
C3	31/30/39(-)	63/24/13(+)	62/29/9(+)	35/23/42(-)	84/7/9(+)	83/7/10(+)	58/31/11(+)	19/78/3(+)
C4	36/29/35(+)	55/25/20(+)	58/24/18(+)	35/20/45(-)	78/2/20(+)	74/6/20(+)	44/29/27(+)	11/84/5(+)
C5	26/35/39(-)	60/25/15(+)	52/30/18(+)	31/26/43(-)	80/8/12(+)	79/9/12(+)	51/32/17(+)	5/93/2(+)
BBOB1	37/57/6(+)	80/19/1(+)	74/24/2(+)	54/42/4(+)	92/7/1(+)	86/11/3(+)	76/21/3(+)	31/58/11(+)
BBOB2	46/51/3(+)	80/16/4(+)	77/21/2(+)	50/42/8(+)	92/6/2(+)	87/11/2(+)	68/27/5(+)	42/52/6(+)
BBOB9	36/49/15(+)	63/25/12(+)	63/25/12(+)	47/35/18(+)	72/21/7(+)	72/18/10(+)	51/29/20(+)	29/59/12(+)
BBOB10	35/42/23(+)	74/18/8(+)	72/18/10(+)	37/36/27(+)	72/15/13(+)	73/16/11(+)	62/18/20(+)	36/34/30(+)

TABLE S.IX  
COMPARATIVE RESULTS OF THE ABLATION STUDY AT GENERATION= $G_{\max}$

Problem	L2T-w/o- $a_1$	L2T-w/o- $a_2$	L2T-w/o- $a_3$	L2T-w/o- $O_c$	L2T-w/o- $O_t$	L2T-w/o- $FE$	L2T-w/o- $r_{conv}$	L2T-w/o- $r_{KT}$
VS	67/26/7(+)	90/7/3(+)	76/22/2(+)	23/50/27(-)	51/47/2(+)	95/4/1(+)	14/58/28(-)	90/7/3(+)
S	47/28/25(+)	86/12/2(+)	76/22/2(+)	26/71/3(+)	39/52/9(+)	85/14/1(+)	19/76/5(+)	84/16/0(+)
M	24/64/12(+)	74/23/3(+)	62/35/3(+)	12/86/2(+)	78/19/3(+)	76/21/3(+)	8/79/13(-)	59/36/5(+)
L	27/59/14(+)	47/47/6(+)	52/43/5(+)	19/74/7(+)	48/48/4(+)	46/49/5(+)	29/59/12(+)	48/37/15(+)
VL	2/83/15(-)	5/88/7(-)	8/89/3(+)	5/89/6(-)	6/88/6(=)	4/91/5(-)	5/92/3(+)	11/84/5(+)
C1	27/64/9(+)	38/55/7(+)	43/52/5(+)	41/55/4(+)	50/48/2(+)	32/59/9(+)	33/63/4(+)	24/63/13(+)
C2	21/71/8(+)	28/66/6(+)	23/70/7(+)	20/78/2(+)	22/72/6(+)	20/72/8(+)	19/69/12(+)	35/54/11(+)
C3	14/72/14(=)	15/77/8(+)	15/81/4(+)	15/82/3(+)	15/84/1(+)	14/79/7(+)	16/81/3(+)	23/70/7(+)
C4	8/88/4(+)	9/87/4(+)	10/88/2(+)	7/91/2(+)	10/87/3(+)	6/93/1(+)	7/90/3(+)	17/79/4(+)
C5	3/90/7(-)	6/88/6(=)	4/92/4(=)	6/86/8(-)	6/86/8(-)	10/86/4(+)	4/92/4(=)	10/84/6(+)
BBOB1	17/78/5(+)	31/59/10(+)	29/58/13(+)	10/87/3(+)	35/51/14(+)	30/60/10(+)	8/87/5(+)	33/57/10(+)
BBOB2	17/74/9(+)	31/58/11(+)	29/65/6(+)	6/91/3(+)	36/55/9(+)	40/52/8(+)	13/85/2(+)	30/58/12(+)
BBOB9	23/72/5(+)	23/60/17(+)	31/61/8(+)	17/75/8(+)	26/63/11(+)	31/55/14(+)	9/79/12(-)	26/63/11(+)
BBOB10	11/68/21(-)	36/38/26(+)	55/36/9(+)	11/65/24(-)	38/39/23(+)	38/36/26(+)	13/68/19(-)	38/40/22(+)

TABLE S.X  
INVESTIGATION RESULTS ON PARAMETER  $b_2$  AT GENERATION= $G_{\text{roll}}$

Problem	$b_2 = 0.1$	$b_2 = 0.5$	$b_2 = 1$	$b_2 = 5$	$b_2 = 10$	$b_2 = 50$	$b_2 = 100$
BBOB1	6.19	6.25	6.02	<b>4.17</b>	4.32	4.62	4.43
BBOB2	6.16	5.98	5.45	<b>3.93</b>	4.39	4.53	4.94
BBOB3	5.90	5.49	5.00	5.82	<b>3.92</b>	4.06	5.30
BBOB4	<b>4.39</b>	5.48	5.44	4.93	5.44	4.91	4.65
BBOB5	<b>4.47</b>	4.81	5.50	5.31	5.02	4.92	4.92
BBOB6	5.21	5.70	<b>4.81</b>	5.05	5.38	5.20	5.15
BBOB7	5.00	4.86	5.22	4.86	5.24	5.12	<b>4.56</b>
BBOB8	4.94	5.32	5.11	5.05	5.06	<b>4.59</b>	5.78
BBOB9	5.50	5.34	5.24	5.06	<b>4.55</b>	4.81	5.20
BBOB10	5.19	5.71	5.26	4.86	<b>4.26</b>	5.02	4.67
BBOB11	6.02	5.62	5.69	4.78	4.40	<b>3.67</b>	5.01
BBOB12	6.14	6.67	5.60	5.32	<b>3.07</b>	4.79	5.03
BBOB13	5.09	5.56	5.85	5.48	<b>4.00</b>	4.46	5.31
BBOB14	4.99	<b>4.90</b>	5.27	4.96	5.37	5.08	5.18
BBOB15	4.92	5.15	5.07	5.72	4.95	4.98	<b>4.90</b>
# best	2	1	1	2	5	2	2

TABLE S.XI  
INVESTIGATION RESULTS ON PARAMETER  $b_2$  AT GENERATION= $G_{\max}$

Problem	$b_2 = 0.1$	$b_2 = 0.5$	$b_2 = 1$	$b_2 = 5$	$b_2 = 10$	$b_2 = 50$	$b_2 = 100$
BBOB1	5.59	5.71	5.98	4.67	4.39	4.50	<b>4.32</b>
BBOB2	6.16	5.72	5.54	4.36	4.19	<b>4.02</b>	5.05
BBOB3	<b>2.98</b>	3.98	5.02	6.00	6.96	7.96	8.96
BBOB4	6.77	7.00	7.07	<b>3.32</b>	3.43	3.59	3.58
BBOB5	4.97	4.82	5.47	<b>4.66</b>	5.27	4.78	4.94
BBOB6	5.55	6.13	4.95	<b>4.45</b>	5.25	4.88	5.14
BBOB7	5.09	4.87	5.27	4.77	5.10	5.60	<b>4.55</b>
BBOB8	5.54	5.24	5.10	5.31	4.76	4.54	<b>4.31</b>
BBOB9	5.66	5.94	5.29	4.61	<b>4.27</b>	4.38	4.74
BBOB10	6.16	6.23	6.26	<b>4.04</b>	4.22	4.08	4.41
BBOB11	5.82	5.66	5.57	4.44	4.78	<b>3.28</b>	4.87
BBOB12	6.62	7.31	6.35	5.79	<b>2.72</b>	4.16	3.70
BBOB13	5.13	5.06	4.95	<b>4.87</b>	5.19	5.04	5.39
BBOB14	4.72	4.91	5.36	5.06	5.34	<b>4.71</b>	5.11
BBOB15	<b>4.85</b>	5.17	4.94	5.72	5.04	5.00	4.94
# best	2	0	0	5	2	3	3

TABLE S.XII  
 THE RESULTS OF MTDE-L2T AND MTGA-L2T COMPARING WITH PEER EMT ALGORITHMS ON HPO PROBLEMS

MTDE-L2T vs						
Problem	AEMTO	MFDE	MKTDE	MTDE-AD	MTDE-B	MTDE-EA
SVM	16/79/5(+)	11/77/12(-)	11/68/21(-)	12/85/3(+)	8/85/7(+)	44/50/6(+)
XGBoost	8/89/3(+)	9/90/1(+)	72/18/10(+)	8/91/1(+)	4/93/3(+)	17/78/5(+)
FCNet	10/88/2(+)	8/91/1(+)	33/53/14(+)	6/93/1(+)	1/94/5(-)	7/91/2(+)
MTGA-L2T vs						
Problem	ATMFEA	GMFEA	MFEA	MFEA-AKT	MFEA2	MTEA-AD
SVM	56/19/25(+)	52/25/23(+)	52/20/28(+)	69/12/19(+)	68/12/20(+)	46/38/16(+)
XGBoost	67/17/16(+)	69/14/17(+)	69/14/17(+)	72/14/14(+)	81/9/10(+)	70/17/13(+)
FCNet	15/61/24(-)	18/73/9(+)	18/71/11(+)	22/66/12(+)	26/68/6(+)	7/78/15(-)