Problem Definitions for cMTOP: A New and More Challenging Compositive Multi-task Optimization Problem Test Suite

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Modified on June 24th 2022

Evolutionary multi-task optimization is a new research area in evolutionary computation. The main idea of evolutionary multi-task optimization is to simultaneously and effectively solve multiple optimization tasks by transferring common knowledge among these tasks. Many evolutionary multi-task optimization algorithms have been proposed and shown promising performance in solving many complex optimization problems.

In recent years, many multi-task optimization problem (MTOP) benchmark test suites have been proposed, including the CEC2017 MTOP benchmark [1], the CEC2019 MTOP benchmark [2], and the WCCI2020 MTOP benchmark test suites [3]. However, most of the MTOPs in these test suites are too simple and have three main disadvantages. First, the tasks in most of these MTOPs have the same dimensionalities. Second, the aligned dimensions of some tasks can have similar physical meanings since they come from the same basic function. Third, the aligned dimensions of the tasks can have similar optimal values. However, these three disadvantages make these MTOPs different from the real-world problems. Specifically, in many real-world problems, the related or similar tasks usually have different numbers of dimensions, and the indexed-based aligned dimensions usually have irrelated physical meanings or have different optimal values.

To address these disadvantages, we propose a new and more challenging compositive MTOP test suite, termed cMTOP. In the cMTOP test suites, there are ten cMTOP instances, each of which has two optimization tasks. The objective function of each task is a compositive function that is composed of at least one basic function. First, the numbers of the dimensions of the two tasks in each cMTOP instance are totally different. Second, the aligned dimensions of the tasks have irrelated physical meanings since each task is composed of several different basic functions and the aligned dimensions come from the same basic function. Third, the aligned dimensions of the tasks have different optimal values since we perform the rotation and shifting operations on each task.

The Matlab codes for the cMTOP test suite can be downloaded from:

https://zhanapollo.github.io/zhanzhh/resources.htm

1. Definitions of the Basic Functions

1) Sphere:

$$F_1(x) = \sum_{i=1}^{D} x_i^2,\ x \in [-100,100]^D \tag{1}$$

2) Rosenbrock:

$$F_2(x) = \sum_{i=1}^{D-1} \left[100 \cdot \left(x_i^2 - x_{i+1} \right)^2 + \left(x_i - 1 \right)^2 \right],\ x \in [-50,50]^D \tag{2}$$

3) Ackley

$$F_3(x) = -20 \cdot \exp\left(-0.2 \cdot \sqrt{\frac{1}{D} \sum_{i=1}^{D} x_i^2} \right) - \exp\left(\frac{1}{D} \sum_{i=1}^{D} \cos\left(2\pi x_i \right) \right) + 20 + e,$$
$$x \in [-50,50]^D \tag{3}$$

4) Rastrigin

$$F_4(x) = \sum_{i=1}^{D} \left(x_i^2 - 10 \cdot \cos\left(2\pi x_i \right) + 10 \right),\ x \in [-50,50]^D \tag{4}$$

5) Griewank

$$F_5(x) = 1 + \frac{1}{4000} \cdot \sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}} \right),\ x \in [-100,100]^D \tag{5}$$

6) Weierstrass

$$F_6(x) = \sum_{i=1}^{D} \left(\sum_{k=0}^{k_{\max}} \left[a^k \cdot \cos\left(2\pi b^k \cdot \left(x_i + 0.5 \right) \right) \right] \right) - D \cdot \sum_{k=1}^{k_{\max}} \left[a^k \cdot \cos\left(2\pi b^k \cdot 0.5 \right) \right]$$
$$a = 0.5, b = 3, k_{\max} = 20,\ x \in [-0.5, 0.5^D] \tag{6}$$

7) Schwefel

$$F_7(x) = 418.9829 \cdot D - \sum_{i=1}^{D} x_i \cdot \sin\left(\left| x_i \right|^{\frac{1}{2}} \right),\ x \in [-500,500]^D \tag{7}$$

2. Introduction to the cMTOP Test Suite

2.1 General Formulation of the Tasks in the cMTOP Test Suite

Each task is a compositive function that is composed of at least one basic function. The general formulation of each task in the cMTOP test suite is defined as follows:

$$f_t(x) = \sum_{i=1}^{N} g_i\left(x(d_i^l : d_i^u) \times M_i^t + S_i^t\right), \;\; g_i \in FS_t \tag{8}$$

where FS_t indicates the set of basic functions involved in the composition function f_t, which contains N basic functions, g_i indicates the i^{th} basic function in FS_t, $x(d_i^l : d_i^u)$ indicates the dimensions of the solution x between the dimension d_i^l and the dimension d_i^u. M_i^t and S_i^t indicate the rotation matrix and the shift matrix of the basic function g_i of the t^{th} task, where the rotation matrix M_i^t is generated according to the method in [1] and the shifting matrix S_i^t is uniformly sampled from a given interval.

To describe the formulation of the compositive function specifically, we take a composition function that is composed of two basic functions as an example. Let f_e stands for the example compositive function, FS_e = [g_1, g_2] stands for the set of basic functions. Suppose the total number of dimensions in f_e is 20, [d_1^l, d_1^u] and [d_2^l, d_2^u] that correspond to g_1 and g_2 are [1, 10] and [11, 20], respectively. That is, the first 10 dimensions correspond to the first basic function g_1 and the last 10 dimensions correspond to the second basic function g_2. The formulation of f_e is given as follows:

$$\begin{aligned} f_e(x) &= \sum_{i=1}^{2} g_i\left(x(d_i^l : d_i^u) \times M_i^t + S_i^t\right) \\ &= g_1\left(x(1:10) \times M_1^t + S_1^t\right) + g_2\left(x(11:20) \times M_2^t + S_2^t\right) \end{aligned} \tag{9}$$

2.2 Definitions and Parameter Settings of the cMTOP Instances

1) cMTOP Problem 1:

Task 1 $N = 1,$

$D = 20,$

$g_1 = \text{Sphere}, [d_1^l, d_1^u] = [1, 20],\ x(d_1^l : d_1^u) \in [-100, 100]^{d_1^u - d_1^l + 1},\ S_1^1 = (0, 0, ..., 0)$

Task 2 $N = 2,$

$D = 50,$

$g_1 = \text{Sphere}, [d_1^l, d_1^u] = [1, 20],\ x(d_1^l : d_1^u) \in [-100, 100]^{d_1^u - d_1^l + 1},\ S_1^2 \in [-40, 40]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Rastrigin}, [d_2^l, d_2^u] = [21, 50],\ x(d_2^l : d_2^u) \in [-50, 50]^{d_2^u - d_2^l + 1},\ S_2^2 = (0, 0, ..., 0)$

where the equation $S_1^2 \in [-40, 40]^{d_1^u - d_1^l + 1}$ indicates that each element in the shift matrix S_1^2 is randomly and uniformly generated in the interval [-40, 40]. The notation $[d_1^l, d_1^u] = [1, 20]$ indicates that d_1^l is set as 1 and d_1^u is set as 20.

2) cMTOP Problem 2:

Task 1 $N = 1,$

$D = 10,$

$g_1 = \text{Rastrigin}, [d_1^l, d_1^u] = [1, 10],\ x(d_1^l : d_1^u) \in [-50, 50]^{d_1^u - d_1^l + 1},\ S_1^1 = (0, 0, ..., 0)$

Task 2 $N = 2,$

$D = 20,$

$g_1 = \text{Griewank}, [d_1^l, d_1^u] = [1, 10],\ x(d_1^l : d_1^u) \in [-100, 100]^{d_1^u - d_1^l + 1},\ S_1^2 \in [-40, 40]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Ackley}, [d_2^l, d_2^u] = [11, 20],\ x(d_2^l : d_2^u) \in [-50, 50]^{d_2^u - d_2^l + 1},\ S_2^2 = (0, 0, ..., 0)$

3) cMTOP Problem 3:

Task 1 $N = 1,$

$D = 50,$

$g_1 = \text{Griewank}, [d_1^l, d_1^u] = [1, 50],\ x(d_1^l : d_1^u) \in [-100, 100]^{d_1^u - d_1^l + 1},\ S_1^1 = (0, 0, ..., 0)$

Task 2 $N = 2,$

$D = 50,$

$g_1 = \text{Schwefel}, [d_1^l, d_1^u] = [1, 25],\ x(d_1^l : d_1^u) \in [-500, 500]^{d_1^u - d_1^l + 1},\ S_1^2 = (0, 0, ..., 0)$

$g_2 = \text{Weierstrass}, [d_2^l, d_2^u] = [26, 50],\ x(d_2^l : d_2^u) \in [-0.5, 0.5]^{d_2^u - d_2^l + 1},\ S_2^2 \in [-0.2, 0.2]^{d_2^u - d_2^l + 1}$

4) cMTOP Problem 4:

Task 1 $N = 1,$

$D = 10,$

$g_1 = \text{Ackley}, [d_1^l, d_1^u] = [1,10],\ x(d_1^l : d_1^u) \in [-50,50]^{d_1^u - d_1^l + 1},\ S_1^1 \in [-20,20]^{d_1^u - d_1^l + 1}$

Task 2 $N = 3,$

$D = 50,$

$g_1 = \text{Sphere}, [d_1^l, d_1^u] = [1,5],\ x(d_1^l : d_1^u) \in [-100,100]^{d_1^u - d_1^l + 1},\ S_1^2 \in [-40,40]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Rastrigin}, [d_2^l, d_2^u] = [6,30],\ x(d_2^l : d_2^u) \in [-50,50]^{d_2^u - d_2^l + 1},\ S_2^2 \in [-20,20]^{d_2^u - d_2^l + 1}$

$g_3 = \text{Weierstrass}, [d_3^l, d_3^u] = [31,50],\ x(d_3^l : d_3^u) \in [-0.5,0.5]^{d_3^u - d_3^l + 1},\ S_3^2 \in [-0.2,0.2]^{d_3^u - d_3^l + 1}$

5) cMTOP Problem 5:

Task 1 $N = 2,$

$D = 40,$

$g_1 = \text{Rosenbrock}, [d_1^l, d_1^u] = [1,20],\ x(d_1^l : d_1^u) \in [-50,50]^{d_1^u - d_1^l + 1},\ S_1^1 = (0,0,...,0)$

$g_2 = \text{Griewank}, [d_2^l, d_2^u] = [21,40],\ x(d_2^l : d_2^u) \in [-100,100]^{d_2^u - d_2^l + 1},\ S_2^1 \in [-40,40]^{d_2^u - d_2^l + 1}$

Task 2 $N = 2,$

$D = 50,$

$g_1 = \text{Griewank}, [d_1^l, d_1^u] = [1,10],\ x(d_1^l : d_1^u) \in [-100,100]^{d_1^u - d_1^l + 1},\ S_1^2 \in [-40,40]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Schwefel}, [d_2^l, d_2^u] = [11,50],\ x(d_2^l : d_2^u) \in [-500,500]^{d_2^u - d_2^l + 1},\ S_2^2 = (0,0,...,0)$

6) cMTOP Problem 6:

Task 1 $N = 2,$

$D = 50,$

$g_1 = \text{Rosenbrock}, [d_1^l, d_1^u] = [1,20],\ x(d_1^l : d_1^u) \in [-50,50]^{d_1^u - d_1^l + 1},\ S_1^1 = (0,0,...,0)$

$g_2 = \text{Schwefel}, [d_2^l, d_2^u] = [21,50],\ x(d_2^l : d_2^u) \in [-500,500]^{d_2^u - d_2^l + 1},\ S_2^1 = (0,0,...,0)$

Task 2 $N = 2,$

$D = 50,$

$g_1 = \text{Ackley}, [d_1^l, d_1^u] = [1,30],\ x(d_1^l : d_1^u) \in [-50,50]^{d_1^u - d_1^l + 1},\ S_1^2 \in [-20,20]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Weierstrass}, [d_2^l, d_2^u] = [31,50],\ x(d_2^l : d_2^u) \in [-0.5,0.5]^{d_2^u - d_2^l + 1},\ S_2^2 \in [-0.2,0.2]^{d_2^u - d_2^l + 1}$

7) cMTOP Problem 7:

Task 1 $N = 2,$

$D = 50,$

$g_1 = \text{Schwefel}, [d_1^l, d_1^u] = [1, 25],\ x(d_1^l : d_1^u) \in [-500, 500]^{d_1^u - d_1^l + 1},\ S_1^1 = (0, 0, ..., 0)$

$g_2 = \text{Sphere}, [d_2^l, d_2^u] = [26, 50],\ x(d_2^l : d_2^u) \in [-100, 100]^{d_2^u - d_2^l + 1},\ S_2^1 \in [-20, 20]^{d_2^u - d_2^l + 1}$

Task 2 $N = 2,$

$D = 50,$

$g_1 = \text{Rastrigin}, [d_1^l, d_1^u] = [1, 25],\ x(d_1^l : d_1^u) \in [-50, 50]^{d_1^u - d_1^l + 1},\ S_1^2 \in [-20, 20]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Ackley}, [d_2^l, d_2^u] = [26, 50],\ x(d_2^l : d_2^u) \in [-50, 50]^{d_2^u - d_2^l + 1},\ S_2^2 \in [-20, 20]^{d_2^u - d_2^l + 1}$

8) cMTOP Problem 8:

Task 1 $N = 2,$

$D = 40,$

$g_1 = \text{Sphere}, [d_1^l, d_1^u] = [1, 20],\ x(d_1^l : d_1^u) \in [-100, 100]^{d_1^u - d_1^l + 1},\ S_1^1 \in [-40, 40]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Ackley}, [d_2^l, d_2^u] = [21, 40],\ x(d_2^l : d_2^u) \in [-50, 50]^{d_2^u - d_2^l + 1},\ S_2^1 \in [-20, 20]^{d_2^u - d_2^l + 1}$

Task 2 $N = 3,$

$D = 50,$

$g_1 = \text{Rastrigin}, [d_1^l, d_1^u] = [1, 20],\ x(d_1^l : d_1^u) \in [-50, 50]^{d_1^u - d_1^l + 1},\ S_1^2 \in [-20, 20]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Griewank}, [d_2^l, d_2^u] = [21, 40],\ x(d_2^l : d_2^u) \in [-100, 100]^{d_2^u - d_2^l + 1},\ S_2^2 \in [-40, 40]^{d_2^u - d_2^l + 1}$

$g_3 = \text{Weierstrass}, [d_3^l, d_3^u] = [41, 50],\ x(d_3^l : d_3^u) \in [-0.5, 0.5]^{d_3^u - d_3^l + 1},\ S_3^2 \in [-0.2, 0.2]^{d_3^u - d_3^l + 1}$

9) cMTOP Problem 9:

Task 1 $N = 3,$

$D = 50,$

$g_1 = \text{Sphere}, [d_1^l, d_1^u] = [1, 20],\ x(d_1^l : d_1^u) \in [-100, 100]^{d_1^u - d_1^l + 1},\ S_1^1 \in [-40, 40]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Ackley}, [d_2^l, d_2^u] = [21, 40],\ x(d_2^l : d_2^u) \in [-50, 50]^{d_2^u - d_2^l + 1},\ S_2^1 \in [-20, 20]^{d_2^u - d_2^l + 1}$

$g_3 = \text{Rosenbrock}, [d_3^l, d_3^u] = [41, 50],\ x(d_3^l : d_3^u) \in [-50, 50]^{d_3^u - d_3^l + 1},\ S_3^2 = (0, 0, ..., 0)$

Task 2 $N = 3,$

$D = 50,$

$g_1 = \text{Rastrigin}, [d_1^l, d_1^u] = [1, 20],\ x(d_1^l : d_1^u) \in [-50, 50]^{d_1^u - d_1^l + 1},\ S_1^2 \in [-20, 20]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Griewank}, [d_2^l, d_2^u] = [21, 40],\ x(d_2^l : d_2^u) \in [-100, 100]^{d_2^u - d_2^l + 1},\ S_2^2 \in [-40, 40]^{d_2^u - d_2^l + 1}$

$g_3 = \text{Weierstrass}, [d_3^l, d_3^u] = [41, 50],\ x(d_3^l : d_3^u) \in [-0.5, 0.5]^{d_3^u - d_3^l + 1},\ S_3^2 \in [-0.2, 0.2]^{d_3^u - d_3^l + 1}$

10) cMTOP Problem 10:

Task 1 $N = 3,$

$D = 50,$

$g_1 = \text{Sphere}, [d_1^l, d_1^u] = [1, 20],\ x(d_1^l : d_1^u) \in [-100, 100]^{d_1^u - d_1^l + 1},\ S_1^1 \in [-40, 40]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Ackley}, [d_2^l, d_2^u] = [21, 40],\ x(d_2^l : d_2^u) \in [-50, 50]^{d_2^u - d_2^l + 1},\ S_2^1 \in [-20, 20]^{d_2^u - d_2^l + 1}$

$g_3 = \text{Rosenbrock}, [d_3^l, d_3^u] = [41, 50],\ x(d_3^l : d_3^u) \in [-50, 50]^{d_3^u - d_3^l + 1},\ S_3^2 = (0, 0, ..., 0)$

Task 2 $N = 4,$

$D = 50,$

$g_1 = \text{Rastrigin}, [d_1^l, d_1^u] = [1, 20],\ x(d_1^l : d_1^u) \in [-50, 50]^{d_1^u - d_1^l + 1},\ S_1^2 \in [-20, 20]^{d_1^u - d_1^l + 1}$

$g_2 = \text{Griewank}, [d_2^l, d_2^u] = [21, 30],\ x(d_2^l : d_2^u) \in [-100, 100]^{d_2^u - d_2^l + 1},\ S_2^2 \in [-40, 40]^{d_2^u - d_2^l + 1}$

$g_3 = \text{Schwefel}, [d_3^l, d_3^u] = [31, 40],\ x(d_3^l : d_3^u) \in [-500, 500]^{d_3^u - d_3^l + 1},\ S_3^2 = (0, 0, ..., 0)$

$g_4 = \text{Weierstrass}, [d_4^l, d_4^u] = [41, 50],\ x(d_4^l : d_4^u) \in [-0.5, 0.5]^{d_4^u - d_4^l + 1},\ S_4^2 \in [-0.2, 0.2]^{d_4^u - d_4^l + 1}$

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